Assignment #1

Prb 2-18, 2-16, 3-2, 3-3

Denavit Hartenburg (DH) representation

A commonly used convention for selecting frames of refrenence in robotics applications.

In this convention, each homogenous transformation A_i is represented as a product of 4 "basic" translations.

$$A_i = R_z(\theta_i)\,Tran_z(d_i)\,Tran_x(a_i)\,R_x(\alpha_i)$$

Note conditions must be ment.

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cdot \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

θ_i : The angle.

a_i : The length

α_i : The twist

d_i : The offset.

Steps for setting up DH

Step 1 Locate and label joint axis, $z_0 \dots z_{n-1}$

Step 2 Establish the base frame. Set the origin anywhere on the z_0 axis. The x_0 and y_0 axis are chosen conviently to form a right hand frame.

For $i=1$ to $i=n-1$ repeat steps 3 to 5

Step 3 Locate the origin O_i where common normal to z_i and z_{i-1} intersects z_i. If z_i intersects z_{i-1} locate O_i at this intersection. If z_i and z_{i-1} are parrel locate O_i at the

Step 4 Establish x_i along the common normal between z_{i-1} and z_i, through O_i

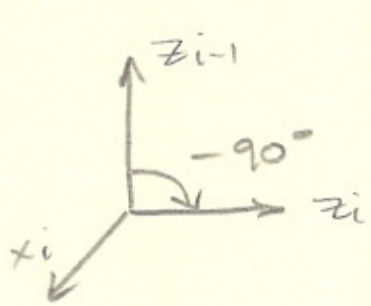
or in the direction normal to the z_{i-1}, z_i plane if z_i and and z_{i-1} intersect.

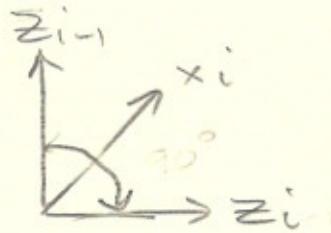
Step 5 Establish y_i to complete a right hand frame

Step 6 Establish the end effector frame o_n, z_n, y_n, x_n.

Note for exam A_i and these steps will be given

Note





Step 7 Create a table of link parameters $a_i, d_i, \alpha_i, \theta_i$.

a_i = distance along x_i

d_i : distance along z_i

α_i : angle b/t z_{i-1} and z_i

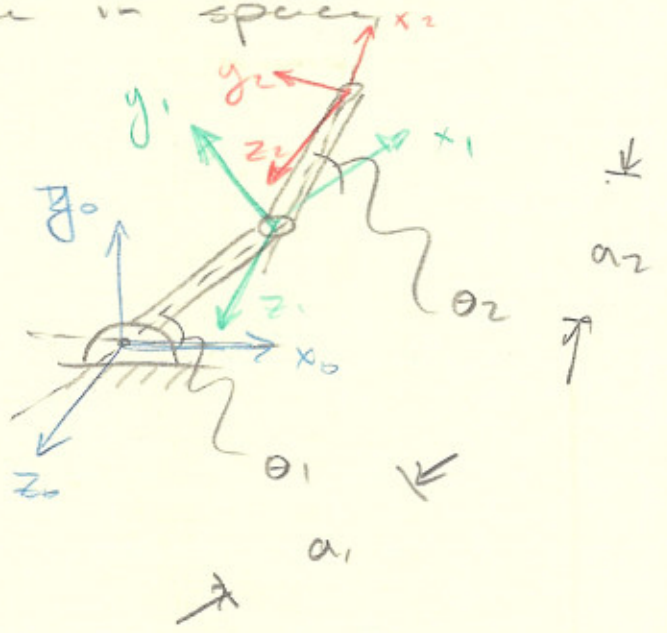
θ_i = angle b/t x_{i-1} and x_i

Step 8 Form the homogenous transformation matrices A_i by substituting the previous parameters.

Step 9 Form $T_0 = A_i \cdots A_n$. This gives the position and orientation of the

Tool frame in space

Ex



∴ table of parameters

link	α	θ	a	d
1	0	θ_1^*	a_1	0
2	0	θ_2^*	a_2	0

$$A_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & a_1 C\theta_1 \\ S\theta_1 & C\theta_1 & 0 & a_2 S\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$$^{0}_{2}H = A_1 \cdot A_2$$

$$= \begin{bmatrix} C_{12} & -S_{12} & 0 & a_1 C_1 + a_2 C_{12} \\ S_{12} & C_{12} & 0 & a_1 S_1 + a_2 S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C_{12} = C_{\theta_1 + \theta_2}$$